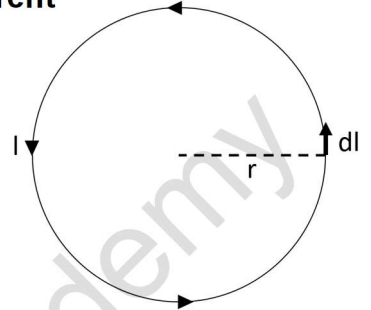


## MAGNETIC EFFECTS OF CURRENT

### ALL DERIVATIONS

#### (1) Magnetic field at the centre of a circular loop carrying current

Consider a circular current carrying loop carrying current  $I$ . We have to find magnetic field at the centre of this loop. Consider a small current element  $dl$  on the circumference of this loop. Clearly angle between  $dl$  and  $r$  is  $90^\circ$ . Applying Biot Savart's law, we get



$$dB = \frac{\mu_0}{4\pi} \left( \frac{Idl \sin 90^\circ}{r^2} \right)$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

Integrating both sides we get

$$\int dB = \int \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

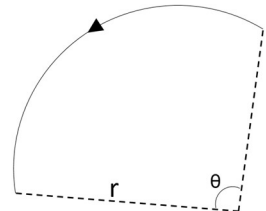
$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int dl$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \times 2\pi r$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2r}}$$

#### (2) Magnetic field due to arc

As complete circle is also an arc which subtends an angle  $2\pi$  at the centre so by applying the unitary method, we can find the magnetic field at the centre of arc as follows:



Angle	Magnetic field
$2\pi$	$\frac{\mu_0 I}{2r}$

---

**1 radian**

$$\left(\frac{\mu_0 I}{2r}\right) \times \frac{1}{2\pi} = \frac{\mu_0 I}{4\pi r}$$

---

**Any angle  $\theta$**

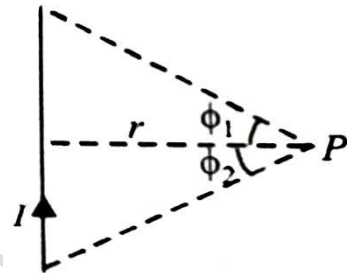
$$B = \frac{\mu_0 I}{4\pi r} \times \theta$$

---

### (3) Magnetic field due to a straight conductor

Magnetic field at point P at a perpendicular distance r from a straight conductor carrying current I is

$$B = \frac{\mu_0 I}{4\pi r} (\sin \phi_1 + \sin \phi_2)$$



#### Special cases

When length of wire is infinite (or very long) and distance r is very small then

- ✓ If P lies near one end, then  $\phi_1 = 90^\circ$  and  $\phi_2 = 0^\circ$

$$\text{so, } B = \frac{\mu_0 I}{4\pi r} (\sin 90^\circ + \sin 0^\circ)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r}$$

- ✓ If P lies near centre, then  $\phi_1 = 90^\circ$  and  $\phi_2 = 90^\circ$

$$\text{so, } B = \frac{\mu_0 I}{4\pi r} (\sin 90^\circ + \sin 90^\circ)$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

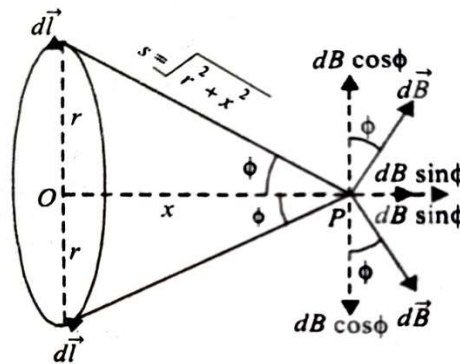
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### (4) Magnetic field on the axis of a circular loop

Small magnetic field due to current element  $Idl$  of circular loop of radius r at a point P at distance x from its centre is

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ}{s^2} = \frac{\mu_0}{4\pi} \frac{Idl}{(r^2 + x^2)}$$

Component  $dB \cos \phi$  due to current element at point P is cancelled by equal and opposite



component  $dB \cos \phi$  of another diagonally opposite current element, whereas the sine components  $dB \sin \phi$  add up to give net magnetic field along the axis. So net magnetic field at point P due to entire loop is

$$\int dB \sin \phi = \int_0^{2\pi} \frac{\mu_0}{4\pi} \frac{Idl}{(r^2 + x^2)^{3/2}} \cdot r$$

$$\Rightarrow B = \frac{\mu_0 Ir}{4\pi(r^2 + x^2)^{3/2}} \int_0^{2\pi} dl$$

$$\Rightarrow B = \frac{\mu_0 Ir}{4\pi(r^2 + x^2)^{3/2}} \cdot 2\pi r$$

$$\Rightarrow B = \frac{\mu_0 Ir^2}{2(r^2 + x^2)^{3/2}}$$

Which is directed along the axis (a) towards the loop if current in it is in clockwise direction  
(b) away from the loop if current in it is in anticlockwise direction.

### Special points

If point P is far away from the centre of the loop i.e.  $x \gg r$  then magnetic field at point P is

$$B = \frac{\mu_0 Ir^2}{2x} = \frac{\mu_0 I \pi r^2}{2\pi x^3} \quad \text{or} \quad B = \frac{\mu_0 IA}{2\pi x^3} \quad \text{where } A \text{ is the area of the circular loop.}$$

If circular loop has N turns then magnetic field strength at its centre is  $B = \frac{\mu_0 NI}{2r}$  and at any

point on the axis of circular loop is  $B = \frac{\mu_0 NI r^2}{2(r^2 + x^2)^{\frac{3}{2}}}$

### (5) Ampere's circuital law

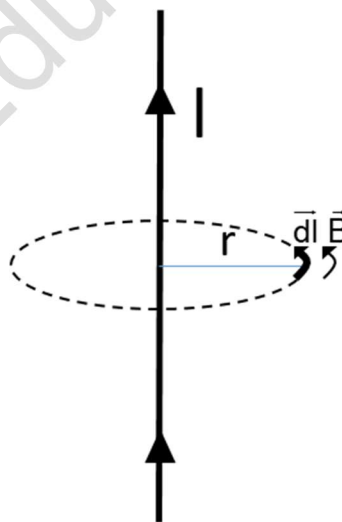
It states that the line integral of magnetic field intensity over a closed loop is  $\mu_0$  times the total current threading the loop.

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$

#### Proof:

Consider a straight conductor carrying current as shown in the figure. Consider a circular Amperian loop of radius r around the conductor. As  $\vec{B}$  and  $d\vec{l}$  are in same direction so angle between them is 0. Therefore

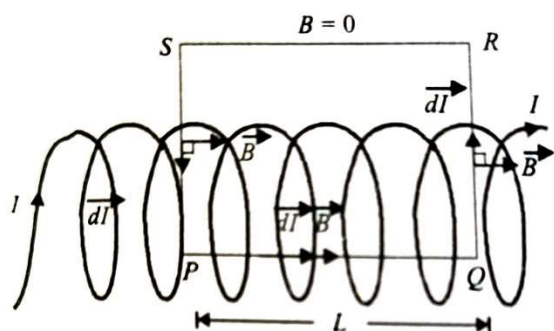
$$\begin{aligned} \int \vec{B} \cdot d\vec{l} &= \int B dl \cos 0^\circ \\ &= \int B dl \\ &= B \int dl \\ &= \frac{\mu_0 I}{2\pi r} \times 2\pi r \\ &= \mu_0 I \\ \therefore \int \vec{B} \cdot d\vec{l} &= \mu_0 I \end{aligned}$$



### Applications of ampere's circuital law

#### (6) Magnetic field intensity at the centre of a long solenoid

Let a solenoid consists of  $n$  no. of turns per unit length and carry current  $I$ . Then magnetic field lines inside the solenoid are parallel to its axis whereas outside the solenoid the magnetic field is zero. Line integral of magnetic field over a closed loop PQRS shown in the figure is



$$\int \vec{B} \cdot d\vec{l} = \int_P^Q \vec{B} \cdot d\vec{l} + \int_Q^R \vec{B} \cdot d\vec{l} + \int_R^S \vec{B} \cdot d\vec{l} + \int_S^P \vec{B} \cdot d\vec{l}$$

$$= \int_P^Q B \cdot dl \cos 0^\circ + \int_Q^R B \cdot dl \cos 90^\circ + 0 + \int_S^P B \cdot dl \cos 90^\circ$$

$$= B \int_P^Q dl + 0 + 0 + 0 = BL$$

But by Ampere's circuital law

$$\int \vec{B} \cdot d\vec{l} = \mu_0 \times \text{total current threading loop PQRS}$$

$$= \mu_0 \times \text{number of turns in solenoid PQRS} \times I$$

$$= \mu_0 nLI$$

Therefore

$$BL = \mu_0 nLI$$

$$\Rightarrow \boxed{B = \mu_0 nI}$$

Note: at the ends of the solenoid the magnetic field is  $\boxed{B = \frac{1}{2} \mu_0 nI}$

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## (7) Force acting on a charged particle moving in a magnetic field

If a charge  $q$  is moving with velocity  $v$  in a magnetic field of intensity  $B$  such that the angle between velocity vector and magnetic field vector is  $\theta$ , then a force  $F$  acts on the particle such that

- i)  $F \propto q$
- ii)  $F \propto v$
- iii)  $F \propto B$
- iv)  $F \propto \sin\theta$

Combining all these, we get

$$F \propto qvB\sin\theta$$

$$\Rightarrow \boxed{F = qvB\sin\theta}$$

As the value of constant in this relation is 1 in SI units.

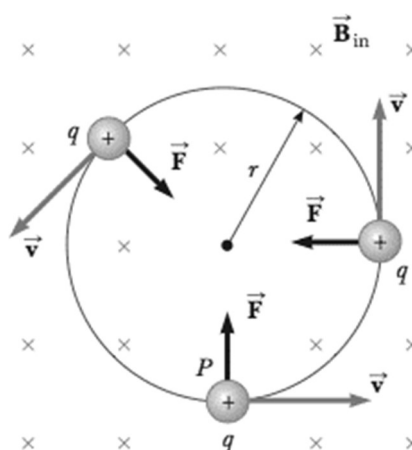
In vector form

$$\vec{F} = q(\vec{v} \times \vec{B}), \text{ thus } F \text{ is perpendicular to the plane containing } v \text{ and } B.$$

If a charge  $q$  enters perpendicularly into a magnetic field, then its path will be circular as force always acts in a direction perpendicular to the direction of motion of the charge. Centripetal force required for circular motion is provided by the magnetic force acting on the particle. Thus

$$\frac{mv^2}{r} = qvB$$

$$\frac{mv}{r} = qB$$



1. **Radius of the path (r)**

$$r = \frac{mv}{Bq}$$

2. **Velocity (v)**

$$v = \frac{Bqr}{m}$$

3. **Time period (T)**

$$T = \frac{2\pi r}{v} = \frac{2\pi \cancel{r}}{\frac{Bq\cancel{r}}{m}} = \frac{2\pi m}{Bq}$$

4. **Frequency**

$$\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

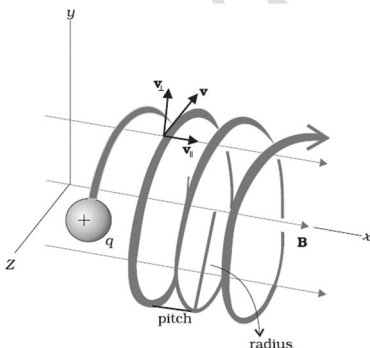
5. **Angular frequency**

$$\omega = 2\pi\nu = 2\pi \times \frac{Bq}{2\pi m} = \frac{Bq}{m}$$

6. **Kinetic energy**

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Bqr}{m}\right)^2$$
$$\Rightarrow KE = \frac{1}{2}m \frac{B^2 q^2 r^2}{m^2} = \frac{1}{2} \frac{B^2 q^2 r^2}{m}$$

If charge particle enters at an angle with the direction of magnetic field then split its velocity into rectangular components  $v \cos \theta$  along the field and  $v \sin \theta$  perpendicular the field as shown. Due to these two components, the motion of the charge is helical.



Distance between two turns of the helix is called **pitch(d)** which is given by

$$d = v \cos \theta \times \text{time period} = v \cos \theta \times \frac{2\pi m}{Bq}$$

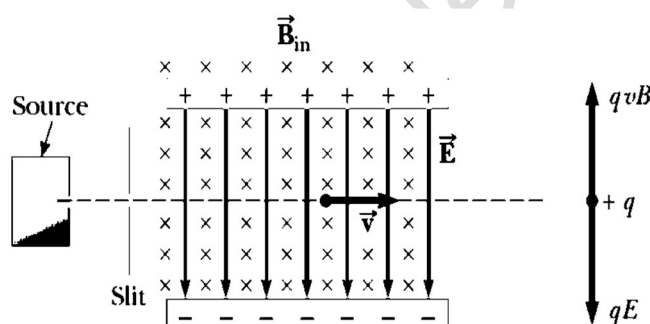
### (8) Velocity selector or velocity filter

Consider a situation as shown in the figure in a charge is moving perpendicularly to both electric and magnetic fields such the force the force acting on charge due to both the fields is equal and opposite i.e.

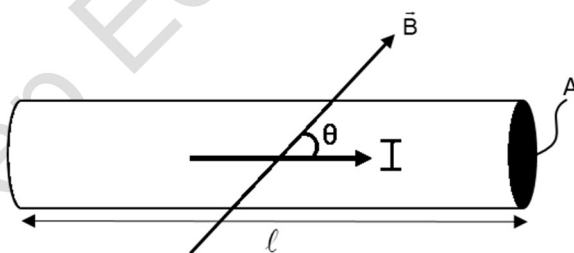
$$qE = qvB$$

$$\therefore v = \frac{E}{B}$$

This result is used in velocity selectors or velocity filters in which we have to select a particle with a particular value of velocity.



### (9) Force acting on a current carrying conductor placed in a magnetic field



Consider a conductor of length  $\ell$  and area of cross section  $A$  carrying current  $I$  placed in a magnetic field at an angle  $\theta$  as shown. If number density of electrons in the conductor is  $n$  then total number of electrons in the conductor is  $A\ell n$ .

As force acting on one electron is  $f = ev_d B \sin \theta$  where  $v_d$  is the drift velocity of electrons.

So the total force acting on the conductor is



$$A\ell n f = A\ell n (ev_d B \sin \theta)$$

$$= (Anev_d) \ell B \sin \theta$$

$$\Rightarrow \boxed{F = I\ell B \sin \theta}$$

Direction of this force can be determined by Fleming's left hand rule.

### (10) Force between two parallel straight conductors carrying current

#### When the currents are in same direction

When two current carrying conductors are placed parallel to each other, each conductor produces a magnetic field around itself. So, one conductor is placed in the magnetic field produced by the other. Using Fleming's left hand rule it can be easily shown that the forces on them are such that they attract each other. Force acting on 1<sup>st</sup> conductor is given as

$$F_1 = I_1 \ell B_2 \sin 90^\circ$$

$$F_1 = I_1 \ell \frac{\mu_0 I_2}{2\pi r}$$

$$\Rightarrow \boxed{\frac{F_1}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

Now force acting on conductor 2 is given by

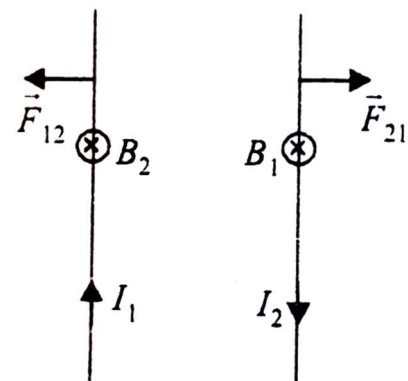
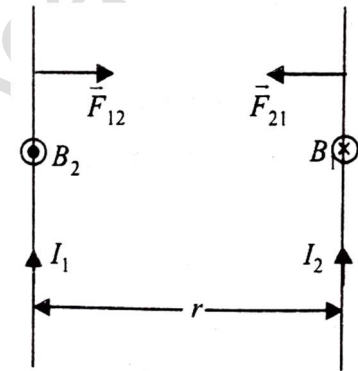
$$F_2 = I_2 \ell B_1 \sin 90^\circ$$

$$F_2 = I_2 \ell \frac{\mu_0 I_1}{2\pi r}$$

$$\Rightarrow \boxed{\frac{F_2}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

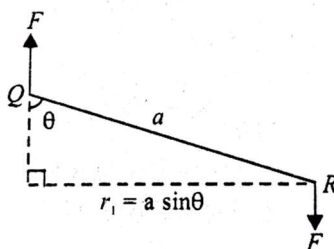
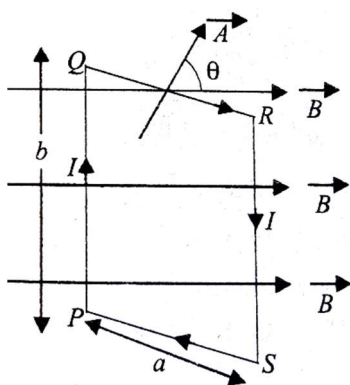
Therefore  $F_1 = F_2$

Note: When the current is in opposite direction the conductors will repel each other the magnitude of force will be same as derived above.



### (11) Torque acting on a current carrying conductor placed in a magnetic field

When a rectangular loop PQRS of sides 'a' and 'b' carrying current I is placed in uniform magnetic field B, such that area vector A makes an angle  $\theta$  with direction of magnetic field, then forces on the arms QR and SP of loop are equal, opposite and collinear, thereby perfectly cancel each other, whereas forces on arms PQ and RS of loop are equal and opposite but not collinear, so they give rise to torque on loop.



Force on side PQ or RS of loop is  $F = I b B \sin 90^\circ = I b B$

Perpendicular distance between two non collinear forces  $r_{\perp} = a \sin \theta$

So, torque on the loop is

$$\tau = F_{\perp} = I b B a \sin \theta = I (ab) B \sin \theta$$

$$\text{or } \boxed{\tau = IAB \sin \theta}$$

If loop has N turns then  $\boxed{\tau = NIAB \sin \theta}$ .

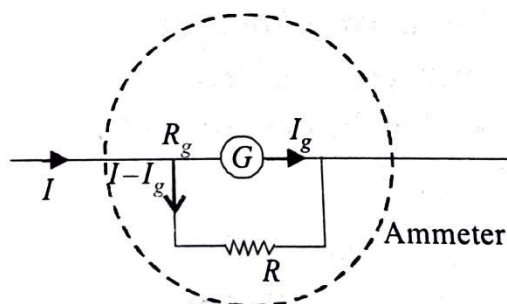
In vector form  $\vec{\tau} = \vec{M} \times \vec{B}$  where  $M = NIA$  is called magnetic dipole moment of current loop and is directed in direction of area vector.

- ✓ If the plane of the loop is normal to the direction of magnetic field i.e.  $\theta = 0^\circ$  between  $\vec{B}$  and  $\vec{A}$  then the loop does not experience any torque i.e.  $\tau_{\min} = 0$
- ✓ If the plane of the loop is parallel to the direction of magnetic field i.e.  $\theta = 90^\circ$  between  $\vec{B}$  and  $\vec{A}$  then the loop experience maximum torque  $\tau_{\max} = NIAB$

---

### (12) Conversion of galvanometer into ammeter

A galvanometer can be converted into ammeter by connecting a low shunt resistance in parallel with it, so that most of the current by passes through the shunt resistance, enabling the galvanometer to measure much larger currents.



Thus if a galvanometer of resistance  $R_g$  which gives full scale deflection at  $I_g$  is to be used to convert into an ammeter capable of measuring a maximum current  $I$ , we connect a shunt resistance  $R$  in parallel with it which is obtained as

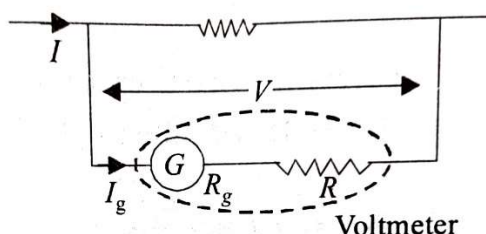
$$V_R = V_G$$

$$\Rightarrow (I - I_g)R = I_g R_g$$

$$\Rightarrow R = \frac{I_g R_g}{I - I_g}$$

### (13) Conversion of galvanometer into voltmeter

A galvanometer can be converted into voltmeter by connecting high resistance in series with it, so that most of the voltage applied drops across it, enabling the galvanometer to measure much larger voltages.



Thus if the galvanometer of resistance  $R_g$  which gives full deflection at current  $I_g$ , is to be converted into voltmeter capable of measuring maximum voltage up to  $V$  volts, then a high resistance  $R$  is connected in series with it which is given by

$$V = I_g R_g + I_g R \text{ or } V - I_g R_g = I_g R$$

or 
$$R = \frac{V}{I_g} - R_g$$

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